The Benefits of Cognitive Mathematics for the Elaboration of Concept Formation Processes

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In his essay "The Psychology of Invention in the Mathematical Field" (1945), Jaques Hadamard pointed out that within the context of educating, learning and thinking of mathematics, "two subjects are involved", i.e. **mathematics** and **psychology**. The approach of pursuing mathematics education from this perspective we call **Cognitive Mathematics**.

The content area of mathematics in nowadays teaching is being extended by computer science/informatics, natural sciences and technology, (labeled MINT in German, STEM in English contexts) all of those having a large potential of making mathematical concepts and connections accessible through their interrelatedness with mathematics.

- Mathematics offers ideas
- Computer science provides space for accessing and handling these ideas
- Natural sciences present application scenarios
- Technology facilitates the production.

As of today, introducing children to mathematics does not yet always succeed in a satisfactory manner (Schwank & Schwank, 2015). This is often due to the fact that Hadamard's perspective is not being completely taken as the basis for action, but also, because human cognition has not yet been sufficiently fathomed. Among the neglected topics in the field of primary arithmetic – a fact that may lead to enormous insurmountable arithmetic difficulties – are:

i. The introduction of the number zero as the initial element of the counting process and the predecessor of the number one
ii. Enhancing of the object view on pluralities with the even more useful process view
iii. Number construction sense as a basic cognitive tool for grasping the number space
iv. Counting backwards on an equal footing with counting forwards
v. Subtraction in equal treatment with and as reverse operation to addition
vi. Introducing the mathematical-formal notation for the plurality that is known as ten in English or sepuluh in Indonesian as the character sequence 10
vii. Understanding the functioning of the decimal number notation
viii. Understanding the calculation operations with numbers in the decimal number notation

ix. Ways of speaking and notation concerning written calculation

Lastly it remains to be said that the area of primary geometry is neglected as a whole, therefore the following large aspect needs to be added:

x. Geometrical conceptions that are more than mere visual perceptions of form, such as with the first simple, plain figures like foursquare, triangle, square, rectangle, circle, ellipse

The development of the mathematical playworlds ENSO (Event-related Number Space Orientation), SSC (Spiral Stairs of Calculation), NUSKY (Number Skyscraper), SAS (Stellanian Accounting System) is oriented closely on Hadamard's program. This lecture will further elucidate this aspect. Furthermore, a learning environment will be introduced that makes simple, plane figures accessible by making the concept of angles a central topic. Along with it the CNC-engine FiloCut (CNC: Computerized Numerical Control) is employed which can be operated by means of computer programs, hereby taking up the ideas of Seymour Papert (1993).

Hadamard, J. (1945). The Psychology of Invention in the Mathematical Field. Princeton: Dover Publication.

Papert, S. (1993): Mindstorms: Children, Computers, and Powerful Ideas. 2nd edition. New York: Basis Books.

Schwank, I. & Schwank, E. (2015): Development of Mathematical Concepts during Early Childhood Across Cultures. International Encyclopedia of the Social & Behavioral Sciences. 2nd edition, Volume 14, 772-784.